Mark scheme for Support Worksheet – Topic 2, Worksheet 5

1 **a** $\frac{F_1}{F_2} = \frac{\frac{GMm}{r_1^2}}{\frac{GMm}{r_2^2}} = \left(\frac{r_2}{r_1}\right)^2 = 4$ [1]

b $\frac{E_1}{E_2} = \frac{-\frac{GMm}{2r_1}}{-\frac{GMm}{2r_2}} = \frac{r_2}{r_1} = 2$ [1]

2 The work done will be equal to the change in the total energy of the satellite; which is $-\frac{GMm}{R+h} - (-\frac{GMm}{R}) = -\frac{GMm}{R+R} - (-\frac{GMm}{R}) = \frac{GMm}{2R}$ [2]

3 The work done will be equal to the change in the total energy of the satellite; which is $-\frac{GMm}{2r_2} - (-\frac{GMm}{2r_1}) = -\frac{GMm}{6R} - (-\frac{GMm}{4R}) = \frac{GMm}{12R}$ [2]

4 **a** At the point of release the total energy of the probe is $E = -\frac{GMm}{r} = -\frac{GMm}{2R}$;

At impact, $-\frac{GMm}{2R} = \frac{1}{2}mv^2 - \frac{GMm}{R} \Rightarrow v = \sqrt{\frac{GM}{R}}$ [2]

b $v = \sqrt{\frac{6.67\times10^{-11}\times6.4\times10^{23}}{3.4\times10^6}} = 3.5\times10^3\ \mathrm{m\,s^{-1}}$ [1]

c It will be much less as Mars has an atmosphere and frictional forces will slow down the probe as it lands. [1]

5 **a** At launch on the surface of the planet the total energy is $E = \frac{1}{2}mv^2 - \frac{GMm}{R}$. At infinity, there is no potential energy and since the escape speed is the minimum speed required, there will be no kinetic energy at infinity also; Hence $E = \frac{1}{2}mv^2 - \frac{GMm}{R} = 0$ from which the result $v = \sqrt{\frac{2GM}{R}}$ follows. [2]

b $v = \sqrt{\frac{2\times6.67\times10^{-11}\times6.0\times10^{24}}{6.4\times10^6}} = 1.1\times10^4\ \mathrm{m\,s^{-1}}$ [1]

c The spacecraft had engines! The formula for escape speed applies only to ballistic motion, i.e. to objects shot like bullets out of a gun. [1]

6 Applying conservation of energy $\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{r}$ with $v = \frac{1}{2}\sqrt{\frac{2GM}{R}}$; so that $\frac{1}{2}m\frac{1}{4}\frac{2GM}{R} - \frac{GMm}{R} = -\frac{GMm}{r}$; i.e. $r = \frac{4R}{3}$ [3]

Copyright Cambridge University Press 2012. All rights reserved.